National Curriculum 2014
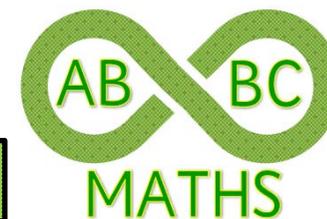
Progression in Written Calculation
St Augustine's Catholic Primary School

The 2014 National Curriculum highlights:

Aims

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of Key Stage 2 to support pupils' conceptual understanding and exploration of more complex number problems if written and mental arithmetic are secure.

5.2 Pupils should be taught to apply arithmetic fluently to problems, understand and use measures, make estimates and sense check their work.
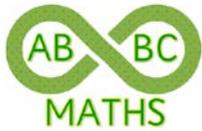
Lower Key Stage 2

The principal focus of mathematics teaching in lower Key Stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

Upper Key Stage 2

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation.

By the end of Year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

AB ∞ BC
MATHS

Progression for WRITTEN ADDITION

AB BC MATHS

Lower Key Stage 2

Year 3

* add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Partition Method

47 + 55

40 and 7
+ 50 and 5
90 and 12 = 102

Also include:

Expanded method to enable conceptual understand using two then three digit numbers

67 + 56

```
  6 7
+ 5 6
-----
  1 3
1 1 0
-----
1 2 3
```

Columnar addition

No exchanges
```
  4 3 5
+ 2 1 3
-------
  6 4 8
```
↓↓↓ ↓↓↓

One exchange
```
  4 3 5
+ 2 1 7
-------
  6 5 2
    ‿
```
↓↓↓ ↓↓↓

Two exchanges
```
  4 3 5
+ 2 8 7
-------
  7 2 2
  ‿‿
```

```
  2 5 1        6 7 9
+   7 3      +   7 3
-------      -------
  3 3 4        7 5 2
    ‿          ‿ ‿
```

Year 4

* add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Columnar addition

No exchanges
```
  3 2 5 1
+ 5 4 1 3
---------
  8 6 6 4
```
↓↓↓ ↓↓↓

One exchanges
```
  3 2 5 1
+ 5 4 7 3
---------
  8 6 3 4
        ‿
```
↓↓↓ ↓↓↓

Two exchanges
```
  2 9 3 8
+ 5 4 2 3
---------
  8 3 6 1
    ‿ ‿
```
↓↓↓ ↓↓↓

Three exchanges
```
  8 9 5 8
+ 5 8 7 3
---------
1 4 7 3 1
  ‿ ‿ ‿
```

Also include:

```
  3 7 5 8        3 7 7 8
+   4 1 3      +   4 8 3
---------      ---------
  4 1 6 1        4 2 6 1
    ‿            ‿ ‿ ‿
```

```
  3 5 1        3 5 5
  2 3 4        2 3 4
+ 4 2 3      + 4 7 3
-------      -------
1 0 0 8      1 0 6 2
    ‿          ‿ ‿
```

Decimal addition in the context of money

Upper Key Stage 2

Year 5

* add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Formal written method

Various exchanges
```
    3 7 2 3 4
+   7 5 4 7 9
-----------
1 1 2 7 1 3
    ‿   ‿ ‿
```

Also include:

```
  2 3 1 8        2 3 . 1 4
      5 3        5 6 0 . 8 3
+   9 2 5      +   4 6 . 7 1
---------      -----------
  3 2 9 6        6 3 0 . 6 8
    ‿            ‿ ‿ ‿
```

Decimal addition in the context of money and measures to 3 d.p

Year 6

* *Pupils practise addition, subtraction … for larger numbers, using the efficient written methods of columnar addition and subtraction*

Formal written method

```
    7 6 2 5 9
    6 8 0 6 8
+     7 5 1 4
-----------
1 5 1 8 4 1
  ‿   ‿ ‿
```

Numbers with different decimal places

5.234 + 43.19 + 387.3

```
      5 . 2 3 4
    4 3 . 1 9 0
+ 3 8 7 . 3 0 0
-------------
  4 3 5 . 7 2 4
  ‿ ‿       ‿
```

Readiness for formal written methods

* know the place value of digits in whole numbers and decimals
* know by heart all addition facts for numbers up to 20
* add at least 3 single digits mentally
* understand zero as a place holder
* add a pair of two digit numbers mentally
* know the approximate size of the answer

Ask yourself:

* Can I do it in my head using a mental strategy?
* Could I use some jottings?
* Should I use a written method?

* **Ensure emphasis is given to the place value of the digits**

Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

Use five of these numbers to make the calculation correct
4, 4, 4, 9, 9, 9
```
  □ □ □
+   □ □
-------
  5 4 8
```

What's the mistake?
```
  1 2 . 3
+   9 . 8
-------
  2 1 . 1 1
```

What's the missing number?
```
  5 4 8
+ 7 □ 7
-------
1 3 2 5
```

* Find two 3-digit numbers with a sum of 465.
* Beth has made a necklace with 123 pink beads and 238 purple beads. How many beads are on the necklace altogether?
* Find the different totals you can make by using any three of these numbers: 1.07, 0.3, 37.03, 17.73, 31.7

Progression for WRITTEN SUBTRACTION

AB BC MATHS

	Lower Key Stage 2		Upper Key Stage 2	
	Year 3	**Year 4**	**Year 5**	**Year 6**

Year 3

* add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Expanded method to enable conceptual understand using two then three-digit numbers

745 - 219

```
       30    15
700 + 40  + 5
- 200 + 10 + 9
-------------------
500 + 20 + 6  = 526
```

↓↓↓ leading to ↓↓↓

Columnar subtraction

No exchanges
```
  5 3 6
- 3 2 1
-------
  2 1 5
```

↓↓↓ ↓↓↓

One exchange
```
  7  7  1
  7  8  4
- 2  3  7
---------
  5  4  7
```

↓↓↓ ↓↓↓

Two exchanges
```
  6 12  1
  7  3  5
- 2  7  8
---------
  4  5  7
```

↓↓↓ ↓↓↓

Including zero
```
  5  9  1
  6  0  3
- 2  4  7
---------
  3  5  6
```

Also include:
```
  5 8 3       4 16  1
- 3 2       - 4  7  4
-------         8  5
  5 5 1     ---------
              3  8  9
```

Year 4

* add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Columnar subtraction

No exchanges
```
  5 8 3 7
- 1 3 2 4
---------
  4 5 1 3
```

↓↓↓ ↓↓↓

One exchange
```
    6  1  1
  4  7  6  7
- 2  3  9  2
-----------
  2  3  7  5
```

↓↓↓ ↓↓↓

Two exchanges
```
  6  1  4  1
  7  5  5  3
- 3  7  2  8
-----------
  3  8  2  5
```

↓↓↓ ↓↓↓

Three exchanges
```
  5  12 1 1
  6  3  2  5
- 1  5  3  6
-----------
  4  7  8  9
```

↓↓↓ ↓↓↓

Including zero
```
  5  9  1
  6  0  4  3
- 4  7  8  1
-----------
  1  2  6  2
```

Also include:
```
  4  1             2  1  4  1
  1 5 3 4        3  1  5  5
-   2 5 4       -    5 3 6
---------       -----------
  1 2 8 0         2 6 1 9
```

Decimal subtraction in the context of money

Year 5

* add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Formal written method

Various exchanges
```
  4  1  5  1
  7  5  3  6  5
- 3  2  5  3  9
-------------
  4  2  8  2  6
```

Also include:
```
  4  1  5  1          1  1  3  1
  7 5 3 6 6        2 0 4 3 9
-       6 2 7     -     5 2 4 7
-------------     -------------
  7 4 7 3 9         1 5 1 9 2
```

Decimal subtraction in the context of money and measures to 3 d.p

Year 6

* *Pupils practise addition, subtraction … for larger numbers, using the efficient written methods of columnar addition and subtraction*

Efficient written method

Numbers with different decimal places

327.5 – 62.63
```
  2  1  16  14 1
  3  2  7 . 5  0
-     6  2 . 6  3
----------------
  2  6  4 . 8  7
```

645.27 – 351.8
```
  5  1  14  1
  6  4  5 . 2  7
- 3  5  1 . 8  0
----------------
  2  9  3 . 4  7
```

Decimal subtraction in the context of money and measures to 3 d.p

Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.
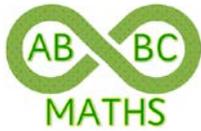
What are the missing digits?
```
  □ 3
- 5 □
-----
  2 5
```

Use the digits 1, 2, 3, 4, 6, 9 to make the calculation correct
```
  □ □
- □ □
-----
  □ □
```

* The Smith family has saved £675 towards their summer holiday. The cost of the holiday is £2019. How much more do they need to save?
* At the beginning of a cricket match there were 742 people watching. At tea-time 218 people went home. How many were left?
* Two numbers have a difference of 1.58. One of the numbers is 4.72. What is the other? Is this the only answer?
* Gordon won £363 630 on the lottery and Betty won £4387, how much more did Gordon win?
* After a sale, Boots cost £55.23 and trainers cost £34.78. How much less do the trainers cost?

Progression for WRITTEN MULTIPLICATION

ABC MATHS

Readiness for formal written methods

* know the place value of digits in whole numbers and decimals
* know by heart all addition facts for numbers up to 20
* add a pair of two digit numbers mentally
* partition a number in different ways
* confident using written addition
* recall and use appropriate multiplication facts
* multiply any positive integer by 10 or a multiple of 10
* understand zero as a place holder
* know the approximate size of the answer

Ask yourself:

* Can I do it in my head using a mental strategy?
* Could I use some jottings?
* Should I use a written method?

Ensure emphasis is given to the place value of the digits

Lower Key Stage 2

Year 3

* write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written

Grid method

23×3

×	20	3
3	60	9	= 69

16×4

×	10	6
4	40	24	= 64

32×8

×	30	2
8	240	16	= 256

Year 4

* multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Grid method

135×6

×	100	30	5
6	600	180	30	= 810

Expanded method to enable conceptual understand

```
    2 4
  ×   3
    1 2
    6 0
    7 2
```

Formal written layout

```
  4 2          3 6
×   3        ×   4
1 2 6        1 4 4
               2
```

```
  3 1 2        2 7 3
×     6      ×     7
1 8 7 2      1 9 1 1
    ₁          ₅ ₂
```

Upper Key Stage 2

Year 5

* multiply numbers up to 4 digits by a one- or two-digit number using an formal written method, including long multiplication for two-digit numbers

Formal written method

```
  2 5 1 3        6 5 7 9
×       7      ×       8
1 7 5 9 1      5 2 6 3 2
  ₃   ₂          ₄ ₆ ₇
```

Long Multiplication

```
    2 7          7 8
×   3 4        × 6 4
  1 0 8        3 1 2
  8 1 0        4 6 8 0
  9 1 8        4 9 9 2
```

```
    1 2 4        2 3 7 4
×     2 6      ×     3 2
    7 4 4        4 7 4 8
  2 4 8 0      7 0 2 2 0
  3 2 2 4      7 4 9 6 8
```

Year 6

* multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
* multiply one-digit numbers with up to two decimal places by whole numbers

Formal written method

```
    6 0 2 7        4 3 7 8
×     3 4        ×     7 3
  2 4 1 0 8      1 3 1 3 4
1 8 0 8 1 0      3 0 6 4 6 0
2 0 4 9 1 8      3 1 9 5 9 4
```

```
  8 . 7          8 . 6 8
×   6          ×     7
5 2 . 2        6 0 . 7 6
```

Also include:

```
  7 8 4 . 9          4 1 . 6 8
×       6          ×       7
4 9 0 9 . 4        2 9 1 . 7 6
```

```
    4 7 . 3          3 1 . 5 6
×     6 2          ×       2 3
    9 4 . 6          9 4 . 6 8
2 8 3 8 . 0      6 3 1 . 2 0
2 9 3 2 . 6      7 2 5 . 8 8
```

Decimal multiplication in the context of money and measures

Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

```
  □ 4 □
×     6
2 0 5 2
```

* There is space in the car park for 17 rows of 32 cars. How many cars can park?
* How many hours are there in one year?
* What is the total mass of 235 screws each weighing 6g?
* Find the area of a swimming pool which is 25m long and 7.5m wide.
* I buy 1.6 kg of apples. They cost 65p per kg. how much do I pay?
* An exercise book is 15mm thick. How thick will a pile of 5 exercise books be?

* How many different answers can be made by using the digits 2, 3 and 4 in this calculation? □ □ x □ =
* Organise the digits 9, 7, 5 and 3 into this calculation to give the greatest possible product □.□ □ x □ =
* Which is closer to 100: 5.2 × 17 or 7.2 × 15? Use written methods to prove your answer
* Abbie says that 23.4 × 5 will be bigger than 53.4 × 2. Is she correct?
* A can of drink contains 0.33 litres. How many litres are in 15 cans?
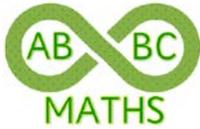
Progression for WRITTEN DIVISION

	Lower Key Stage 2		Upper Key Stage 2	

Lower Key Stage 2

Year 3

Pupils develop reliable written methods for … division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of … short division.

Partitioning

39 ÷ 3

$$3 \overline{)\begin{array}{c} 1\ 0\ +\ 3 = 13 \\ 3\ 0\ +\ 9 \end{array}}$$

↓↓↓ ↓↓↓

64 ÷ 4

$$4 \overline{)\begin{array}{c} 1\ 0\ +\ 6\ =\ 16 \\ 4\ 0\ +\ 24 \end{array}}$$

↓↓↓ ↓↓↓

72 ÷ 3

$$3 \overline{)\begin{array}{c} 2\ 0\ +\ 4\ =\ 24 \\ 6\ 0\ +\ 12 \end{array}}$$

Year 4

Pupils practise to become fluent in the formal written method of … short division with exact answers

Partitioning

119 ÷ 7

$$7 \overline{)\begin{array}{c} 1\ 0\ +\ 7\ =\ 17 \\ 7\ 0\ +\ 49 \end{array}}$$

216 ÷ 9

$$9 \overline{)\begin{array}{c} 2\ 0\ +\ 4\ =\ 24 \\ 1\ 8\ 0\ +\ 36 \end{array}}$$

Short Division

Divisor $3 \overline{)\begin{array}{c} 2\ 1 \\ 6\ 3 \end{array}}$

↓↓↓ ↓↓↓

$6 \overline{)\begin{array}{c} 1\ 4 \\ 8\ ^2 4 \end{array}}$ Quotient

Dividend

Upper Key Stage 2

Year 5

- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Short Division

$$7 \overline{)\begin{array}{c} 2\ 3 \\ 1\ 6\ ^21 \end{array}} \qquad 8 \overline{)\begin{array}{c} 3\ 4 \\ 2\ 7\ ^32 \end{array}}$$

$$6 \overline{)\begin{array}{c} 2\ 4\ 1 \\ 1\ 4\ ^24\ 6 \end{array}} \qquad 7 \overline{)\begin{array}{c} 1\ 3\ 4\ 5 \\ 9\ ^24\ ^31\ ^35 \end{array}}$$

There are 421 children here today. How many teams of 9 can we make?

$$9 \overline{)\begin{array}{c} 4\ 6\ r\ 7 \\ 4\ 2\ ^61 \end{array}}$$ = 46 teams

206 tickets were sold for a concert; there are 7 seats per row, how many rows are needed?

$$7 \overline{)\begin{array}{c} 2\ 9\ r\ 3 \\ 2\ 0\ ^66 \end{array}}$$ = 30 rows

Year 6

- divide numbers up to 4 digits by a two-digit number using the formal written method of short division …
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, …
- use written division methods in cases where the answer has up to two decimal places

Formal written method

$$1\ 1 \overline{)\begin{array}{c} 4\ 5 \\ 4\ 9\ ^55 \end{array}} \qquad 2\ 1 \overline{)\begin{array}{c} 2\ 4 \\ 5\ 0\ ^84 \end{array}}$$

$$1\ 2 \overline{)\begin{array}{c} 3\ 6\ 1 \\ 4\ 3\ ^73\ ^12 \end{array}}$$

```
              1 3 2
      2 6 ) 3 4 3 2
           - 2 6 0 0    x100
               8 3 2
             -  7 8 0    x30
                 5 2
             -   5 2     x2
                  0
```

```
             2 6 . 4
     1 5 ) 3 9 6 . 0
          - 3 0 0      x20
              9 6
            -  9 0      x6
               6 . 0
             - 6 . 0    x0.4
                0
```

Also include:

$$6 \overline{)\begin{array}{c} 5\ 7\ .\ 2\ 6 \\ 3\ 4\ ^43\ .\ ^15\ ^36 \end{array}}$$

Readiness for formal written methods

- know the place value of digits in whole numbers and decimals
- know by heart all addition and subtraction facts for numbers up to 20
- partition a number in different ways
- recall appropriate multiplication facts
- use known facts and place value to multiply and divide mentally
- confident using written subtraction
- understand zero as a place holder
- know the approximate size of the answer

Ask yourself:

- Can I do it in my head using a mental strategy?
- Could I use some jottings?
- Should I use a written method?

Ensure emphasis is given to the place value of the digits

Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

- Work out whether or not 29 is a factor of 811
- How many 35p packets of stickers can I buy with £5?
- Coaches have 56 seats for passengers. How many coaches are needed to take 275 people on a trip?
- Pencils come in packs of 12. How many packs does a school need to buy to get 310 pencils?
- My mobile phone costs 18p per minute for national calls. If I put £5 on my card, how many minutes can I talk for?
- The area of a rectangular games hall is 384 square metres. If the length is 24 metres, how wide is it?

- Work out what the missing digit for 37☐ ÷ 17, when there is a remainder of 5.
- I bought some pencils that cost 15p each. I paid £5.85. How many pencils did I buy?
- Four children collected £19 for charity. They each collected the same amount. How much was this?
- Rupert saves the same amount of money each month. He saved £149.40 in a year. How much money does he save each month?
- Three bags of crisps weigh 130.5g, how much does one bag weigh?